10/3 Recitation Solutions

1. Let $\mathbf{v} \in \mathbb{R}^n$ and $c \in \mathbb{R}$. Show that $||c\mathbf{v}|| = |c|||\mathbf{v}||$.

   Let $\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$. Then

$$||c\mathbf{v}|| = \left|\left| c \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \right|\right| = \left|\left| \begin{bmatrix} cv_1 \\ \vdots \\ cv_n \end{bmatrix} \right|\right| = \sqrt{(cv_1)^2 + \cdots + (cv_n)^2}$$
$$= \sqrt{c^2v_1^2 + \cdots + c^2v_n^2}$$
$$= \sqrt{c^2(v_1^2 + \cdots + v_n^2)}$$
$$= \sqrt{c^2}\sqrt{v_1^2 + \cdots + v_n^2}$$
$$= |c|||\mathbf{v}||$$

2. Let V be any subspace of $\mathbb{R}^n$. Show that the zero subspace $\{\mathbf{0}\}$ is orthogonal to V.

   Since $\mathbf{0} \cdot v = 0$ for all $v \in V$, $\mathbf{0}$ is orthogonal to every vector of V. Hence every vector in the zero subspace (i.e. $\mathbf{0}$) is orthogonal to every vector of V. That is, $\{\mathbf{0}\}$ is orthogonal to V.

3. Let $V = \text{Span}\{\mathbf{v}, \mathbf{w}\}$ be a subspace of $\mathbb{R}^n$. Show that if $\mathbf{u}$ is orthogonal to $\mathbf{v}$ and $\mathbf{w}$, then it is orthogonal to V (that is, it is orthogonal to all vectors in V).

   Let $\mathbf{x} = c_1\mathbf{v} + c_2\mathbf{w} \in V$ be an arbitrary vector in V. Since $\mathbf{u}$ is orthogonal to $\mathbf{v}$ and $\mathbf{w}$, $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w} = 0$. Hence $\mathbf{u} \cdot \mathbf{x} = \mathbf{u} \cdot (c_1\mathbf{v} + c_2\mathbf{w}) = c_1\mathbf{u} \cdot \mathbf{v} + c_2\mathbf{u} \cdot \mathbf{w} = c_1(0) + c_2(0) = 0$; that is, $\mathbf{u}$ is orthogonal to $\mathbf{x}$. Since $\mathbf{x}$ is arbitrary, we have shown that $\mathbf{u}$ is orthogonal to all vectors in V.

4. Let $V = \text{Span}\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 14 \end{bmatrix} \right\}$ and $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ a \end{bmatrix} \right\}$. Determine the values of a (if any) that makes $V \perp W$. What is going on geometrically?

   In order for V to be orthogonal to W, we need $\begin{bmatrix} 1 \\ 2 \\ a \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$ and $\begin{bmatrix} 1 \\ 2 \\ a \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \\ 14 \end{bmatrix} = 0$. The first equation gives $3a - 3 = 0$ or $a = 1$ and the second gives $14a + 5 = 0$ or

$a = -\frac{5}{14}$. Since these cannot be simultaneously satisfied, W is never orthogonal to V. Geometrically, this means the line given by W is not perpendicular to the plane given by V.

5. Open the **Julia8DotProducts** notebook to see relevant dot product commands. Then complete the "Coding Exercises" at the bottom of the notebook.